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Nonaxisymmetric instability of rapidly rotating black hole in five dimensions

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We present results from numerical solution of Einstein's equation in five dimensions describing evolution of rapidly rotating black holes. We show, for the first time, that the rapidly rotating black holes in higher dimensions are unstable against nonaxisymmetric deformation; for the five-dimensional case, the critical value of spin parameter for onset of the instability is ≈ 0.87.

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I Introduction: Black holes (BHs) are the most strongly self-gravitating objects in nature and also the simplest celestial objects, described by a small number of parameters. Therefore, BHs are expected to reflect the natures of gravitational theories in the clearest manner. This fact motivates extensive studies of BHs not only for four-dimensional (4D) spacetime (see, e.g., [1] for a historical review) but also for higher-dimensional one (see, e.g., [2] for a review).

Since a possibility of BH formation in particle accelerators was pointed out, studies for BHs in higher-dimensional spacetimes have been accelerated. If our space is a 3-brane in large [3] or warped [4] extra dimensions, the Planck energy could be of $O(\mathrm{TeV})$ that may be accessible with huge particle accelerators like the Large Hadron Collider (LHC). In the presence of the extra dimensions, mini BHs may be produced in the accelerators and its evidence may be detected.

A hypothetical phenomenology in the accelerators is as follows [5, 6]: In a high-energy particle collision of a sufficiently small impact parameter, two particles merge to form a distorted rotating BH, and then, it perhaps relaxes to a quasistationary state after emission of gravitational waves. Then, the BH will evaporate by Hawking radiation due to a quantum-field-theory effect in a curved spacetime. The BH formation and subsequent evolution by gravitational radiation can be described by classical general relativity [11]. Since any approximation breaks down for this phase because of its highly nonlinear nature, numerical simulation in full general relativity is the unique approach (see [7, 8, 9, 10] for the 4D case).

One of the most important issues to be clarified is what type of a black object is formed and whether it is stable or not. In the 4D case, the formed object has to be a Kerr BH because of the uniqueness theorem of BHs (e.g., [12] for a review), and the numerical analysis of a Kerr BH strongly suggests that it should be stable [13]. These facts strongly constrain the possible scenarios for the BH formation and subsequent evolution. By contrast, there is no uniqueness theorem in higher dimensions: In the five-dimensional (5D) case, the black ring solution with the ringlike horizon is known in addition to the Myers-Perry BH solution with the spherical horizon (but see [14] for uniqueness of 5D black holes with the spherical horizon topology). Even if we assume that the black rings are unlikely to be formed in two-particle systems, the scenario of mini BHs at accelerators is still uncertain because there is no proof for the stability of the higher-dimensional BHs [2].

Higher-dimensional BHs with certain parameters are known to be unstable against axisymmetric perturbations. Emparan and Myers [15] suggested that rotating BHs with a high spin parameter is unstable for the spacetime dimensionality $D \geq 6$. The reason is that the rapidly rotating BHs have a high degree of ellipticity (i.e., the black membrane limit) and such objects are subject to the Gregory-Laflamme instability [16]. Very recently, Dias *et al.* indeed showed, by a linear perturbation analysis, that rapidly rotating BHs for $7 \leq D \leq 9$ are unstable against axisymmetric multiple-ring-like deformation [17].

On the other hand, little is known for the stability of BHs against the *nonaxisymmetric* perturbation, and also, for the dimensionality $D = 5$. Emparan and Myers [15] discussed the possibility that the rapidly rotating BHs may be unstable also against nonaxisymmetric perturbation for $D \geq 5$ using a thermodynamical argument (i.e., by comparing the horizon area of a rotating BH and that of two boosted Schwarzschild BHs with the same total energy and angular momentum). However, the correspondence between the thermodynamical and dynamical instabilities has not been well established, and thus, a rigorous analysis is required in order to clarify whether the rapidly rotating BHs are actually unstable against nonaxisymmetric perturbation or not.

In this paper, we tackle this stability issue of rapidly rotating 5D BHs by fully solving Einstein's equation. The merits of this approach are that a wide variety of instabilities can be clarified with no ambiguity, and that the final fate after the onset of the instabilities could be determined since the amplitude of the perturbation from the background BH solution need not be assumed to be small. In this letter, we focus on the 5D BHs of only one spin parameter because such a BH will be an outcome in the particle accelerators. We shall explicitly show, for the first time, that rapidly rotating 5D BHs are unstable against nonaxisymmetric distortion.

II Setting and Methodology: We numerically study the stability against nonaxisymmetric deformation of a 5D rotating BH of single spin parameter. Its line element

in the Boyer-Lindquist-type coordinates is [18]

$$ds^2 = -dt^2 + \frac{\mu}{\Sigma}(dt - a\sin^2\theta d\varphi)^2 + \frac{\Sigma}{\Delta}d\hat{r}^2 + \Sigma d\theta^2 + (\hat{r}^2 + a^2)\sin^2\theta d\varphi^2 + \hat{r}^2\cos^2\theta d\chi^2, \quad (1)$$

where μ and a are mass and spin parameters, respectively, and $\Sigma := \hat{r}^2 + a^2\cos^2\theta$ and $\Delta := \hat{r}^2 + a^2 - \mu$. $\hat{r} = r_h := (\mu - a^2)^{1/2}$ is the radius of event horizon. In five dimensions, $q := |a|/\mu^{1/2}$ has to be less than the unity (i.e., $|a| < \mu^{1/2}$) by the requirement of global hyperbolicity. Note that we adopt the geometric units $G = 1 = c$ throughout this letter.

We evolve this BH adopting the so-called quasi-isotropit coordinate r defined by $\hat{r} = r + r_h^2/4r$. Here, the location of the horizon is $r = r_h/2$. With this transformation, the $t =$ const. hypersurface becomes spacelike everywhere for $0 \le r < \infty$, and the physical singularity is not included in the initial surface $t = 0$. In this coordinate, the event horizon corresponds to the wormhole throat and, the sphere denoted by $r = 0$ represents spacelike infinity of another asymptotically flat region and becomes a coordinate singularity. However, the puncture approach of numerical relativity (with the appropriate choice of the conformal factor and gauge conditions) enables us to stably follow the evolution of BHs not only for 4D spacetimes [22] but also for 5D ones [20].

Einstein's equation for a 5D vacuum spacetime are solved in the higher-dimensional version of the BSSN (Baumgarte-Shapiro-Shibata-Nakamura) formalism [19, 20] with a fourth-order finite differencing scheme in space and time. All the equations are solved in the $4+1$ form with the Cartesian coordinates (x, y, z, w) where w denotes the coordinate of the extra dimension. The following so-called puncture gauge conditions are adopted for the lapse function α and shift vector β^i,

$$\partial_t \alpha = -1.5\alpha K, \quad (2)$$

$$\partial_t \beta^i = \eta_B B^i, \quad \partial_t B^i = \partial_t \tilde{\Gamma}^i - \mu^{-1/2} B^i, \quad (3)$$

where B^i an auxiliary gauge variable, K the trace part of the extrinsic curvature, $\tilde{\Gamma}^i$ is an auxiliary variable for the BSSN formalism, and we choose $\eta_B = 1/3$.

For this BH, rotational motion exists only in the (x, y)-plane. Because we consider the nonaxisymmetric stability against a bar deformation, the z- and w-axes directions are equivalent. Thus, a rotational Killing vector $(\partial/\partial\chi)^\mu$ is present where $\chi = \tan^{-1}(w/z)$. This fact motivates us to adopt a cartoon method [20, 21] that enables to solve 4+1 Einstein's evolution equation in the 3D grid (x, y, z) (see [20]).

Numerical simulation was performed with two codes. One is the code reported in [20]. This code prepares a nonuniform grid for which the region around the BH is resolved with a high accuracy and the distant region is covered by a sparse grid spacing. The other is the SACRA5D code in which an adaptive mesh refinement algorithm is implemented. This is the code extended from the SACRA code which was originally developed for simulations of 4D spacetimes [23]. SACRA5D has been tested by solving the problems listed in [20]. Several simulations were performed for both codes and we have confirmed that the two codes provide the same conclusion concerning the stability of BHs.

Although the numerical solutions show a behavior of convergence with improving grid resolution, the convergence speed depends strongly on the spin parameter, q. In this work, we measured the accuracy by monitoring the area of the apparent horizon by evolving nonperturbed rotating BHs and determined the required resolution: We evolved the BHs at least for $50\mu^{1/2}$ and checked required resolution with which the horizon area remains approximately constant within 1% error. For $q \lesssim 0.7$, we found that the grid spacing near horizon $\Delta x \approx 0.03\mu^{1/2}$ is small enough for this requirement. However, for $q \gtrsim 0.8$, the required value for Δx changes steeply: For $q = 0.8$ with $\Delta x/\mu^{1/2} = 0.015$ and 0.0225, the errors at $t = 50\mu^{1/2}$ are 0.2% and 1.2%, respectively. For $q = 0.85$ and 0.88, $\Delta x/\mu^{1/2}$ should be smaller than ≈ 0.01 and 0.008, respectively, to guarantee the error within 1% at $t = 50\mu^{1/2}$. For $q \ge 0.89$, accurate simulation up to $t = 50\mu^{1/2}$ is not feasible even for $\Delta x = 0.008\mu^{1/2}$. Therefore, for this case, the simulations were performed up to the time at which numerical error of apparent horizon area exceeds 3% error (at $t < 50\mu^{1/2}$) choosing $\Delta x = 0.008\mu^{1/2}$. Even these short-term simulations are long enough to show that rapidly rotating BHs are unstable.

To investigate the nonaxisymmetric stability, we initially superimpose a small nonaxisymmetric perturbation on the rotating BH solution. Specifically, we perturb a conformal factor of the 4D space defined by $W = \det(\gamma_{ij})^{-1/4}$ (γ_{ij} is the 4D space metric) as

$$W = W_0[1 + A\mu^{-1}(x^2 - y^2)\exp(-r^2/2r_h^2)], \quad (4)$$

where W_0 is the nonperturbed solution. In the following, we choose $A = 0.005$. Simulations were also done with values $A = 0.01$ and 0.02, but the evolution of the perturbed part obeys a scaling relation (e.g., $W/(W_0 A)$ behaves approximately in the same manner). Thus, the magnitude of A does not change the conclusion in this paper, as far as $A \ll 1$.

During numerical simulation, we monitor two quantities for determining the stability against the nonaxisymmetric deformation. One is a deformation parameter of BH horizon. To define this parameter, we calculate the circumferential radii of the apparent horizon along several meridians. Specifically, we measure the proper length of the meridians (referred to as l_φ) for $\varphi = 0$ (and π), $\pi/4$ (and $5\pi/4$), $\pi/2$ (and $3\pi/2$), and $3\pi/4$ (and $7\pi/4$), and then, define the deformation parameter

$$\eta = \sqrt{(l_0 - l_{\pi/2})^2 + (l_{\pi/4} - l_{3\pi/4})^2}/l_0 \quad (5)$$

The other is the gravitational waveform in a wave zone defined along the z-axis by

$$h_+ \equiv r^{3/2}\mu^{-3/4}(\tilde{\gamma}_{xx} - \tilde{\gamma}_{yy})/2, \quad (6)$$

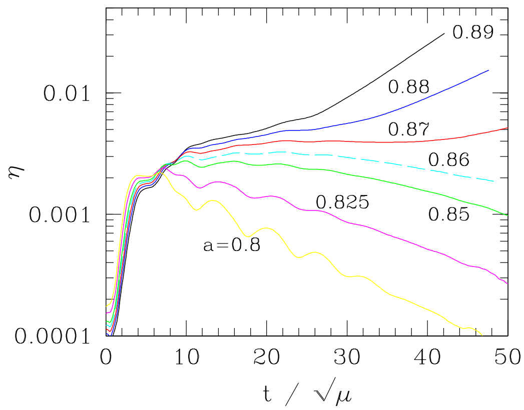

FIG. 1: Evolution of deformation parameter η for $a/\mu^{1/2} =$ 0.80–0.89.

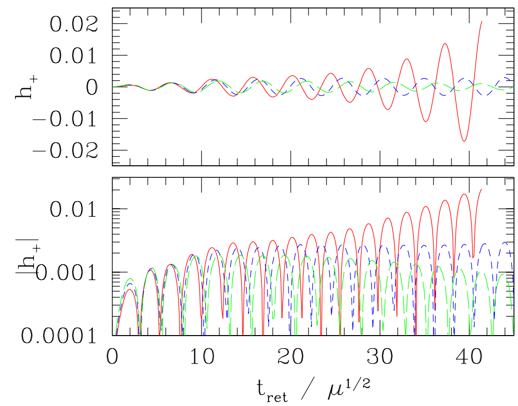

FIG. 2: h_+ and its absolute value as functions of retarded time for $a/\mu^{1/2} = 0.85$, 0.87, and 0.89 (dashed, long-dashed, and solid curves). h_+ is extracted for $r \sim \lambda$ where λ is a gravitational-wave length.

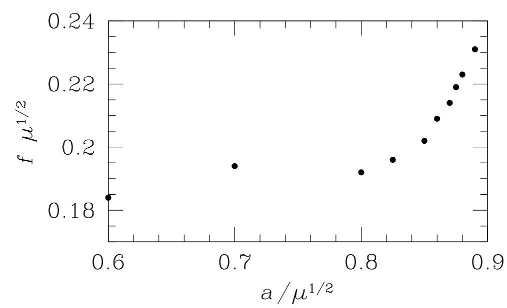

FIG. 3: Characteristic frequency of gravitational waves as a function of $a/\mu^{1/2}$.

where $\tilde{\gamma}_{ij}$ is the conformal 4D metric. This quantity is regarded as the $+$ mode of gravitational waves.

In the 4D case, the amplitude of these quantities decreases exponentially with time as far as $q < 1$, because the BHs are stable; the damping rate is primarily determined by the fundamental quasinormal modes. However, this is not the case in the 5D case (see below).

III Results: Figure 1 plots the evolution of η as a function of time for $q = 0.8$–0.89. Irrespective of the values of q, η quickly increases for $t \lesssim 10\mu^{1/2}$. This is simply because the perturbation initially given relaxes. Indeed, the value of η after this relaxation phase is of $O(10^{-3})$, which is the same order as the initial perturbation amplitude. However, the evolution after the relaxation depends strongly on the values of q. For $q \leq 0.86$, the value of η decreases; in particular for $q \leq 0.80$, this damping occurs very quickly. By contrast, for $q \gtrsim 0.88$, the perturbation grows exponentially for $t/\mu^{1/2} \gtrsim 25$. The growth rate does not depend on the initial perturbation amplitude. Thus, the BHs of $q \gtrsim 0.88$ are dynamically unstable against the nonaxisymmetric deformation. For $q = 0.87$, the value of η grows slowly with time. This suggests that this value is near the critical value for the onset of this instability.

Figure 2 plots h_+ and $|h_+|$ as a function of a retarded time $t_{\rm ret} \equiv t - r$. The amplitude for $t \gtrsim 25\mu^{1/2}$ exponentially increases with time for $q = 0.89$ as in Fig. 1.

From gravitational waveforms, it is possible to extract the frequency of gravitational waves, f. Figure 3 plots a characteristic frequency of gravitational waves as a function of q. Here, the characteristic frequency is determined by performing the Fourier transformation of $h_+(t)$ and then by identifying its peak. Figure 3 shows that f increases with q for $q \geq 0.8$. This property agrees with that of the fundamental quasinormal mode for 4D Kerr BHs [24]. Furthermore, the frequency determined for $a = 0$ gives $f \approx 0.15/\mu^{1/2}$ which agrees with that obtained by a perturbation analysis [25]. Hence, it is natural to consider that emitted gravitational waves are associated with the quasinormal modes of the BHs.

A remarkable fact is that the angular velocity defined by $\omega := \pi f$ is smaller than that of BHs $\Omega = a/(r_h^2 + a^2) = a/\mu$ for a large value of a. Remember that the total radiated energy (ΔE) and angular momentum (ΔJ) approximately obey a relation $\omega \Delta J = \Delta E$, and the first law of the BH thermodynamics allows us to obtain the variation in the BH area δA as $\kappa \delta A / 8\pi = \Omega \delta J - \delta E$ where κ is the surface gravity of the BH horizon, and δE and δJ are the variation of the energy and angular momentum of the BH. If δE and δJ are equal to ΔE and ΔJ, respectively, we obtain the relation

$$\kappa \delta A = 8\pi(\Omega - \omega)\delta J. \tag{7}$$

For $\Omega > \omega$, δA becomes positive, and thus, the evolution by emission of gravitational waves is allowed: A rapidly rotating BH in five dimensions may be unstable against gravitational radiation reaction (although this is not a sufficient condition). This fact seems to be at least a part of the reason why rapidly rotating BHs are unstable against nonaxisymmetric deformation.

IV Summary: We showed, for the fist time, that rapidly rotating BHs in 5D vacuum spacetime are unstable against nonaxisymmetric deformation. The critical value for the spin parameter is ≈ 0.87. The critical value we find is close to the value predicted by Emparan and Myers, ≈ 0.85 [15]. This suggests that their heuris-

tic thermodynamic argument concerning the stability of higher-dimensional rotating BHs may be reliable.

Unfortunately, the present numerical simulation cannot clarify the final fate of the unstable BHs, because it is not easy to maintain the numerical accuracy to follow the unstable BH for a sufficiently long time. To clarify the final fate, a simulation with a much better grid resolution is required. This issue is left for the future study.

There will be at least two possible fates for an unstable BH. One is that the perturbation grows until the unstable BH fragments into two BHs [15], and the other is that the growth of the perturbation saturates at a stage when the emission rate of gravitational waves is large enough to quickly carry angular momentum of the BHs for stabilization. This issue is quite similar to the nonaxisymmetric dynamical stability of rotating stars in four dimensions: For many cases, rotating stars are dynamically unstable if the ratio of rotational kinetic energy to gravitational potential energy is larger than 0.27 or the ratio of the polar axial length to the equatorial axial length is smaller than ~ 0.2 [26]. This condition holds irrespective of equations of state, as far as the degree of differential rotation is not extremely large (e.g., [27] and references therein). Dynamically unstable rotating stars evolve after the onset of the instability via several mechanisms such as angular momentum transfer and gravitational radiation reaction. Then, they settle to a new stable state. A noteworthy fact is that fragmentation rarely occurs for stars (for tori, it may occur).

For the 5D BHs, the ratio of the meridional circumferential length to the equatorial one decreases with q, and $C_m/C_e = 0.38$ for $q = 0.87$. Thus, a BH is unstable for $C_m/C_e \lesssim 0.38$. For the 4D case, the minimum value of this ratio is at most $C_m/C_e = 0.64$ (for the extreme Kerr BH); all the BHs are not very oblate. This may be the reason that BHs are stable in four dimensions. These facts suggest that a sufficiently oblate BH with $C_m/C_e \lesssim 0.4$ is dynamically unstable against nonaxisymmetric deformation irrespective of the dimensionality, as in rapidly rotating stars. Rapidly rotating BHs in any higher dimensions can have small values of C_m/C_e [2], and thus, this issue should be also explored by numerical-relativity simulation.

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